

Will the \mathcal{PT} -symmetric and Non-Hermitian ϕ^4 Theory Solve the Hierarchy and Triviality Problems in the Standard Model?

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Abstract

Very recently, the Large Hadron Collider was turned on. There, the experiments are aiming to test different scenarios for elementary particles interactions from SUSY, Extra dimensions to others. In fact, SUSY was invented to kill the conceptual problems existing in the conventional Standard model *i.e.* the Hierarchy problem. However, in recent years, certain theories which was rejected in the past like the wrong sign ϕ^4 theory is now well known to be a truly physically acceptable theory. Here, we analyze the renormalization group flow of the different parameters in the theory. We find that, rather than the conventional ϕ^4 theory and because the theory is asymptotically free, it does not sufferer from the catastrophic blow up of the mass parameter for UV scales. This feature greatly recommend that this theory is a plausible candidate to play the role of the $SU(2) \times U(1)$ symmetry breaking in the standard model. If this picture will agree with the experimental results from LHC, then the Higgs particle was massless in the far past. Moreover, the cosmological constant problem as a Hierarchy problem may be solved too. However, when trying to calculate the metric operator, we expected that the equivalent Hermitian theory may attain non-renormalizable terms and thus the theory is incalculable. In fact, we show that there exists a Hermitian and renormalizable equivalent theory though of indefinite metric. We reformulated the Klein-Gordon equation in such a way that the wrong sign in the propagator attains a new physical meaning that the Higgs particle is made from exotic matter.

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The origin of mass of the building blocks of our universe impresses the nowadays physicists [1]. The reason behind that is the existence of conceptual problems in the Standard model. Such conceptual problems occur because of our ultimate hope to unify all the existent forces. The unification of the electromagnetic, weak and strong interactions introduces the energy scale 10^{16} GeV , the scale at which the three couplings be the same (in the supersymmetry (SUSY) regime). However, we have another two interesting energy scales, the electroweak scale and the Planck scale. In fact, the scalar Higgs does possess two problems regarding the flow of the parameters either for small or high energy scales. For low energy scales, the theory is trivial while if one tried to let the dimensionful parameters to flow to a very high energy scales, one gets big values for those parameters which will violate the observation. The big values for the dimensionful parameters are well known as gauge Hierarchy problem.

There exists more than one algorithm suggested to solve the Hierarchy problem. For instance, SUSY has been introduced with the aim to have natural cancellation in the dimensionful parameters that turned those parameters protected against perturbations even for very high energy scales [2]. However, it introduces an upper limit to the Higgs mass by 130 GeV and some of its mass spectra are of one TeV which make this theory under the fire of the LHC experiments test by 2008. Another algorithm for the solution of the Hierarchy problem is to consider the Higgs particle as a composite state bound by a new set of interactions (Technicolors)[3]. However, the technicolor model is strongly constrained from precision tests of electroweak theory at LEP and SLC experiments [4]. Also, this algorithm has mass spectra of about one TeV and it is under the direct test of the LHC experiments. Once more, a recent algorithm is suggested for which the $SU(2) \times U(1)$ symmetry is broken via the compactification of an extra dimension. In fact, particles in this model attain masses through the expectation value of the fifth (for instance) component of the gauge field. However, to some physicists, the digestion of extra dimension is not that easy and can be accepted to them at most as a mathematical modeling to the problem. In this letter, we analyze a modified algorithm which we think it can solve the famous triviality as well as the Hierarchy problems. In this algorithm, we revisit a previously rejected theories because they are non-Hermitian but in view of the recent discoveries of the reality of some class of non-Hermitian models (\mathcal{PT} -symmetric), there exists no reason to prevent them from playing in the scene. We think that this is fair and the final word about the correct scenario may come from the analysis of the experimental data from the LHC.

In 1998, Carl Bender and Stefan Boettcher have shown that a class of Non-Hermitian but \mathcal{PT} -symmetric Hamiltonians have real spectra [5]. This discovery led us to reinvestigate the non-Hermitian quantum field models like the $(-g\phi^4)$ and $(g\phi^4 + h\phi^6)$ scalar theories [6, 7]. In the first model, we realized that, rather than the corresponding Hermitian model $(g\phi^4)$, the vacuum energy is tiny for a wide range of energy scales. Besides, the vacuum energy is real and in fact, it is easy to show that the PT symmetry is exact which verify that not only the lowest energy but the full energy spectrum is real [8]. However, what makes this field theory very impressive is that it is asymptotically free [9, 10, 11]. Moreover, we discovered that the Hermitian model $(g\phi^4 + h\phi^6)$ can have a transition to a phase for which the theory is physically acceptable though non-Hermitian [7]. In fact, we asserted that this model has a very interesting property namely, matter-antimatter asymmetry, which is the first time to be obtained in a spontaneously symmetry breaking regime. Relying on such interesting properties of \mathcal{PT} -symmetric and non-Hermitian models, one may ask if such new subject can help in solving the above mentioned problems in the standard model. In this letter, we answer this question by renormalization group analysis of the $(-\frac{\lambda}{4!}\phi^4)$ scalar field model in $3 + 1$ dimension. In fact, the idea we relied on is that the theory is asymptotically free and conclusions drawn from this model can be generalized to the more reliable complex scalar field (doublets) that is used to break the $SU(2) \times U(1)$ symmetry in the standard model.

In the ϕ^4 model in $3 + 1$ dimension, which is used to break the $SU(2) \times U(1)$ symmetry in the standard model, up to one loop, we realize that the mass term receives a correction of the form;

$$M_H^2 = M_0^2 + \frac{3\lambda\Lambda^2}{8\pi^2}, \quad (1)$$

where the M_H^2 and M_0^2 are the renormalized and the bare mass squared of the Higgs particle while λ is the coupling constant. In fact, the appearance of the momentum cutoff Λ is the reason behind the Hierarchy problem, which leads to the introduction of the supersymmetry where the cutoff Λ from a Boson and a Fermion loops cancels. Instead of the SUSY additive cancellation, one can guess a multiplication softening of the Hierarchy problem. By this we mean, if we have an asymptotically free Higgs particle, when Λ is very high, the coupling is very small and thus one may expect that the perturbative correction $\frac{3\lambda\Lambda^2}{8\pi^2}$ stays small. To test this idea, consider the renormalization group functions of the \mathcal{PT} -symmetric and non-Hermitian $(-\frac{\lambda}{4!}\phi^4)$ scalar field model in $3 + 1$ dimensions up to second order in the

coupling;

$$\beta(\lambda) = -\frac{3\lambda^2}{(4\pi)^2}, \quad (2)$$

$$\gamma_m(\lambda) = \frac{-\lambda}{(4\pi)^2} - \frac{5}{6} \left(\frac{\lambda}{(4\pi)^2} \right)^2, \quad (3)$$

where $\beta(\lambda) = \mu \frac{d\lambda}{d\mu}$ and $\gamma_m(\lambda) = \frac{\mu}{m} \frac{dm}{d\mu}$ are the well known renormalization group functions for the flow of the coupling and the mass parameters. Accordingly, the mass parameter can be given by;

$$m^2(\mu) = m^2(\mu_0) \exp \left(\int_{\lambda_0}^{\lambda_\mu} \frac{\gamma_m(\lambda)}{\beta(\lambda)} d\lambda \right) \quad (4)$$

$$= m^2(\mu_0) \exp \left(-\frac{1}{288} \frac{-5\lambda_\mu - 96 \left(\ln \frac{\lambda_\mu}{\lambda_0} \right) \pi^2 + 5\lambda_0}{\pi^2} \right). \quad (5)$$

In fact, because $\beta(\lambda)$ is negative it will drag the coupling to the origin as μ goes to higher values. This behavior is well known as the asymptotic freedom. Accordingly, the Higgs mass will go also to very small values at high energy scales. Thus, if this picture is the successful one in view of the coming analysis from LHC, not only was the quarks, leptons, vector Bosons were massless in the far past but also the Higgs particle was massless. To make the difference between the features of the Hermitian ϕ^4 and the \mathcal{PT} -symmetric ϕ^4 more illustrative, we plotted the behavior of the coupling constant as a function of the logarithm of the mass scale μ in Fig. 1 and Fig.2, respectively. One can easily realize from the figures that the \mathcal{PT} -symmetric ϕ^4 theory is non-trivial and asymptotically free while the Hermitian ϕ^4 is both trivial and the coupling blows up for UV scales which causes the hierarchy problem. In fact, the main message we want to spread in this letter is that (i) there is no gauge Hierarchy problem with the non-Hermitian and \mathcal{PT} -symmetric Higgs mechanism, provided that the contribution to the renormalization group functions from other sectors in the standard model will not affect the asymptotic freedom property of the scalar sector (ii) the model is non-trivial (iii) the technical problem concerning the remedy of the indefinite metric of the theory in the conventional Hilbert space may be solved via a simple Bogoliubov transformation for which the new representation is Hermitian and thus the theory is unitary though of indefinite norm which we will try to attribute it to the existence of a new physical meaning. In fact, it seems that we are obligated to follow that

route as the existing regimes for handling the \mathcal{PT} -symmetric theories introduces a metric operator of the form $\eta_+ = \int d^3x e^{-Q(x)}$ which is expected to introduce non-renormalizable terms in the equivalent Hermitian theory, in case we are able to calculate the metric operator for this theory in $3 + 1$ dimensions [24]. We will investigate this point later on in this work.

Now, we may speculate about if this is the correct picture, why we did not discovered the Higgs particle yet? The answer to this question may be that, in this picture the Higgs particle is a strongly interacting particle and one can not isolate a single Higgs, the same way of behavior of quarks and gluons. Then, one may instead talk about bound states of Higgs which we call it Higgs balls (like glue balls). Thus, according to this picture, the discovery of the Higgs is not a matter of building bigger and bigger machines for the sake of higher and higher energies but a matter of our understanding of the nature of the Higgs particle.

Although of the above mentioned amazing features of the non-Hermitian and \mathcal{PT} -symmetric ϕ^4 theory toward the solution of the genuine problems in the standard model, there exists a technical problem concerning the expected negative norm of the theory. In fact, in the Hilbert space with the Dirac sense inner product operation, the theory have a positive norm but unitarity is violated. This led Bender *et.al* to suggest the PT inner product of the form [12];

$$\langle A|B\rangle_{PT} = (PT|A\rangle)^T|B\rangle.$$

This suggestion preserves unitarity but the Hilbert space with the PT inner product has an indefinite metric. Again, this led Bender *et.al.* to introduce what is called the C operator and the Hilbert space with the CPT inner product preserves unitarity as well as having a positive definite metric. However, the calculation of the C operator for the model under consideration is not that easy and will turn the computation cumbersome. Although this is a technical problem and not a conceptual one, up to the best of our knowledge, no body has obtained the C operator for the non-Hermitian and \mathcal{PT} -symmetric ϕ^4 theory. However, another (equivalent) regime to cure the indefinite metric problem has been suggested by Mostafazadeh [13]. What is important to us here from the Mostafazadeh regime is that the non-Hermitian form of the Hamiltonian is nothing but another representation of an equivalent Hermitian representation and one can (in principle) switch between the two representations via a similarity transformation. However, the metric operator in the

Mostafazadeh regime is hard to get in the non-real line theories especially in quantum field models. In fact, both Bender and Mostafazadeh regimes will lead to a dynamical Feynmann rules in the sense that the Feynmann rules will change from order to order because of the new operators introduced to the theory in the definition of the positive definite inner product. Accordingly, one may ask if there exist a simple similarity transformation by which one can obtain an equivalent Hermitian theory and thus having a Hilbert space with the conventional Dirac sense inner product. We will try to answer this question in the following part of the work.

The reality of the spectrum of a non-Hermitian \mathcal{PT} -symmetric Hamiltonian demands an exact PT symmetry in the sense that all the wave functions respect the PT symmetry in the same footing as the Hamiltonian itself. In the above discussions, we did not check if the states have an exact PT symmetry or not. To show that, let $|n(k)\rangle$ is a state consisting of $n(k)$ identical particles with momentum k . Because the field is assumed to transform as a pseudo scalar under PT transformations we get the transformation of the creation and annihilation operators as;

$$PTa(PT)^{-1} = PT \left(i \int dx \left(\exp(-ikx) \pi - \phi \left(\overleftrightarrow{\frac{\partial}{\partial t}} \right) \exp(ikx) \right) \right) (PT)^{-1} = -a, \quad (6)$$

where $\overleftrightarrow{\left(\frac{\partial}{\partial t}\right)}(AB) = A \frac{\partial B}{\partial t} - \frac{\partial A}{\partial t} B$.

Also, $PTa^\dagger(PT)^{-1} = -a^\dagger$.

Since,

$$|n(k)\rangle = \frac{a^{\dagger n(k)}(k)|0\rangle}{\sqrt{n(k)!}}, \quad (7)$$

$$PT|n(k)\rangle = (-1)^{n(k)} |n(k)\rangle, \quad (8)$$

where we observe that \mathcal{PT} -symmetry is broken. To keep the PT symmetry unbroken, we add the famous extra i^n factor to the states in the following way;

$$|n(k)\rangle = \frac{i^{n(k)} a^{\dagger n(k)}(k)|0\rangle}{\sqrt{n(k)!}}, \quad (9)$$

$$PT|n(k)\rangle = |n(k)\rangle. \quad (10)$$

Also, a state consisting of many particles of different momenta, can be represented by

$$|n(k_1)n(k_1)n(k_3)n(k_4)\dots n(k_m)\rangle = \prod_{j=1}^{j=m} \frac{i^{n(k_j)} a^{\dagger n(k_j)}(k_j)|0\rangle}{\sqrt{n(k_j)!}}, \quad (11)$$

$$PT|n(k)\rangle = |n(k)\rangle. \quad (12)$$

In this way one can build up states which are \mathcal{PT} -symmetric too and thus the \mathcal{PT} symmetry is exact which is an essential requirement for the reality of the \mathcal{PT} -symmetric Hamiltonian.

Now consider the Hamiltonian model of the form;

$$H = H = \frac{1}{2} ((\nabla\phi)^2 + \pi^2 + m^2\phi^2) - \frac{\lambda}{4!}\phi^4,$$

where π is the conjugate momentum. Making the field shift $\phi = \psi + B$; one can rewrite the Hamiltonian density in the form

$$H = H_0 + H_1 + H_I + E, \quad (13)$$

where

$$\begin{aligned} H_0 &= \frac{1}{2} ((\nabla\psi)^2 + \Pi^2 + M^2\psi^2), \\ H_1 &= \left(m^2 - \frac{\lambda}{6}B^2\right) B\psi, \\ H_I &= \frac{-\lambda}{4!} (\psi^4 + 4B\psi^3), \\ E &= \frac{1}{2}m^2B^2 - \frac{\lambda}{4!}B^4, \end{aligned}$$

where B is the vacuum condensate, $\Pi = \dot{\psi}$ and $M^2 = m^2 - \frac{\lambda}{6}B^2$. A well known condition for the effective potential E is to satisfy the equality $\frac{\partial E}{\partial B} = 0$. Accordingly, the term H_1 is ought to be zero. Thus, the quasi-particle Hamiltonian in Eq.(13) has the form

$$H = \int d^3x \left(\frac{1}{2} ((\nabla\psi)^2 + \Pi^2 + M^2\psi^2) - i \frac{\lambda|B|}{6}\psi^3 - \frac{\lambda}{4!}\psi^4 \right), \quad (14)$$

where we used the fact that the vacuum condensate of this theory is pure imaginary [6, 14]. Accordingly, the theory is non-Hermitian but \mathcal{PT} -symmetric and thus is physically acceptable. However, one of the essential problems opposing this theory is that the metric operator has not been obtained so far. In fact, the form in Eq.(14) enables us to apply the conventional tools to calculate its metric operator at least in a perturbative way. To show

this consider consider the non-Hermitian Hamiltonian in Eq.(14). Mostafazadeh generalized the requirement of real spectra for a non-Hermitian theory to the existence of a positive definite metric operator η such that $\eta_+ H \eta_+^{-1} = H^\dagger$ [15, 16] with an equivalent hermitian theory h such that

$$\rho H \rho^{-1} = h \text{ with } \rho = \sqrt{\eta} = \exp\left(-\frac{Q}{2}\right) \text{ where } \eta = \exp(-Q). \text{ Accordingly, we can get}$$

$$\begin{aligned} H^\dagger &= \exp(-Q) H \exp(Q) = H + [-Q, H] + [-Q, [-Q, H]] \\ &\quad + [-Q, [-Q, [-Q, H]]] + \dots \end{aligned}$$

Also, one has a similar expansion for the Hermitian Hamiltonian $h = \exp(\frac{-Q}{2}) H \exp(\frac{Q}{2})$, which will result in a perturbative expansion for h as

$$h = h_0 + g h_1 + g^2 h_2 + \dots$$

Now, we have for H^\dagger the expansion;

$$\begin{aligned} \exp(-Q) H \exp(Q) &= H_0 + g H_I + [-Q, H_0] + [-Q, g H_I] + [-Q, [-Q, H_0]] + \\ &\quad [-Q, [-Q, g H_I]] + [-Q, [-Q, [-Q, H_0]]] + [-Q, [-Q, [-Q, g H_I]]] \dots \\ &= H_0 + g H_I^\dagger, \end{aligned}$$

with

$$Q = Q_0 + g Q_1 + g^2 Q_2 + g^3 Q_3 + \dots$$

Thus, we get a set of coupled equations for the operators Q_n , where the first few equations

are given by

$$\begin{aligned}
0 &= [-Q_0, H_0] \Rightarrow Q_0 = 0 \text{ is a good choice,} \\
H_I^\dagger - H_I &= -2ig \int d^3x \psi^3 = -\frac{1}{2}[-Q_1, H_0], \\
0 &= \frac{1}{2}[-Q_2, H_0] + \frac{1}{2}[-Q_1, H_I] + \frac{1}{3!}[Q_1, [Q_1, H_0]], \\
0 &= \frac{1}{2}[-Q_3, H_0] + \frac{1}{2}[-Q_2, H_I] + \frac{1}{3!}[Q_2, [Q_1, H_0]] \\
&\quad + \frac{1}{3!}[Q_1, [Q_2, H_0]] + \frac{1}{4!}[-Q_1, [-Q_1, [-Q_1, H_0]]] \\
&\quad + \frac{1}{3!}[-Q_1, [-Q_1, H_I]], \\
0 &= \frac{1}{2}[-Q_4, H_0] + \frac{1}{4}[-Q_3, H_I] + \frac{1}{3!}[-Q_2, [-Q_2, H_0]] \\
&\quad + \frac{1}{5!}[Q_1, [Q_1, [Q_1, [Q_1, H_0]]]] + \frac{1}{3!}[-Q_2, [-Q_1, H_I]] \\
&\quad + \frac{1}{3!}[-Q_1, [-Q_2, H_I] + \frac{1}{4!}[-Q_1, [-Q_1, [-Q_1, H_I]]]] \\
&\quad + \frac{1}{8 \times 4!}[-Q_1, [-Q_1, [-Q_2, H_0]]]] \\
&\quad + [-Q_1, [-Q_2, [-Q_1, H_0]]]] + [-Q_2, [-Q_1, [-Q_1, H_0]]]].
\end{aligned} \tag{15}$$

In fact, this regime has been used before to calculate the Q operator for a Hamiltonian form that is similar to the effective form of the $-\frac{\lambda}{4!}\phi^4$ theory in Eq.(13) in 0 + 1 dimensions [17]. However, one can expect that the equivalent Hermitian theory is non-renormalizable. Moreover, following the work in Ref.[18], one can obtain the Q operator up to first order for the more simpler $i\phi^3$ theory which will take the form;

$$Q_1 = \int \int \int d^3x d^3y d^3z \left(M_{(xyz)} \Pi(x) \Pi(y) \Pi(z) + N_{x(yz)} \psi(y) \Pi(x) \psi(z) \right),$$

where the functions $M_{(xyz)}$ and $N_{x(yz)}$ are defined there. However, the resulting Hermitian Hamiltonian h have terms for which the coupling has a negative mass dimension and the situation will be worst in higher orders as more powers of both the fields Π and ψ are appearing in the operator Q and thus in the Hermitian Hamiltonian h . In other words, the transformation $\rho H \rho^{-1} = h$ does not respect the superficial degree of divergence [19]. In fact, this was the reason that leads the authors of Ref.[20] to have a Hermitian theory which is incalculable equivalent to the calculable non-Hermitian ix^3 theory. Relying on these analysis, we expect that this is will be the case for the $-\phi^4$ theory and the calculation of the metric operator which depends on the field and its conjugate momentum is thus meaningless.

Now, we have a theory (the $-\lambda\phi^4$ theory) which shows up interesting behaviors like asymptotic freedom and it seems that it is free from the hierarchy problem. However, the theory seems to be incalculable as well. To escape from this puzzle one can seek another metric operator for which the Hamiltonian is pseudo-Hermitian and leading to an equivalent Hermitian form though with a wrong sign propagator. In fact, one may gauge the parity operator. To do that, we take into account that the quasi-particle field ψ is pseudo scalar and thus

$$PHP^{-1} = H^\dagger,$$

where P is the parity operator. Then, one can introduce the operator ρ such that $P = \rho^2$. In this case, we have the following transformation properties

$$\rho\psi\rho^{-1} = -i\psi, \quad \rho\Pi\rho^{-1} = i\Pi.$$

Note that both P and ρ preserve the commutation relations of the field operators $[\psi(x), \Pi(y)] = i\delta^3(x - y)$. In view of this, one can obtain

$$h = \rho H \rho^{-1} = H = \int d^3x \left(\frac{-1}{2} ((\nabla\psi)^2 + \Pi^2 + M^2\psi^2) - i\frac{\lambda|B|}{6}\psi^3 - \frac{\lambda}{4!}\psi^4 \right). \quad (16)$$

Note that h is Hermitian but the propagator has an opposite sign to the conventional ϕ^4 theory. Moreover, all the Feynman diagrams calculated either with h or H have the same numerical value as it should be. Also, both h and H are normalizable theories rather than the expected Hermitian Hamiltonian obtained from $\exp(-Q)H\exp(Q)$, with Q is a functional in ψ and Π fields and calculated from the coupled set in Eq.(15). Now, the Hamiltonian has ghost states. However, one can attribute this to a new physical meaning. To show this, consider the propagator $\frac{-i}{p^2 - M^2}$, in multiplying by M we obtain a new propagator of the form

$$\frac{-iM}{p^2 - M^2} = \frac{-i}{\frac{p^2}{M} - M},$$

which can be considered as the matter probability amplitude with the new hypothesis that matter density can be negative or positive. Accordingly, the wrong sign appears in the propagator can be attributed to a particle of negative mass *i.e.* made of exotic matter, provided that the jump from the non-relativistic quantum mechanics to the relativistic quantum mechanics has done via $\frac{p^2}{2M} \rightarrow \frac{p^\mu p_\mu}{2M}$. Or equivalently, have the Klein-Gordon equation of the form;

$$\left(\frac{\nabla^2 - \frac{\partial^2}{\partial t^2}}{2m} - \frac{m}{2}\right)\psi = 0,$$

which is the same for both positive and negative values of m . In the presence of interactions, there exists two different Klein-Gordon equations one for positive m and another one for negative m . However, for a negative mass particle, the quantum field Hamiltonian for the theory under consideration have the form;

$$h = \rho H \rho^{-1} = H = \int d^3x \left(\frac{-1}{2} \left(\frac{(\nabla\psi)^2 + \Pi^2}{m} + m\psi^2 \right) - i\frac{\lambda|B|}{6}\psi^3 - \frac{\lambda}{4!}\psi^4 \right), \quad (17)$$

and in this way the negative sign can attributed to a theory of negative mass. Note that, this form is Hermitian and thus the theory is unitary in the Dirac sense inner product.

In conclusion, we showed that the non-Hermitian and \mathcal{PT} -symmetric ϕ^4 theory has very interesting features as an asymptotically free theory. The most important feature of the theory is that the parameters of the theory are perturbative for UV scales rather than the corresponding Hermitian theory used to break the $SU(2) \times U(1)$ symmetry in the standard model. This suggests that using the negative coupling ϕ^4 theory instead, will solve many problems in the standard model. Out of this problems, is that the negative coupling ϕ^4 is not trivial as it has interactions allover the energy scale because of the asymptotic freedom property. Also, it might save the standard model if the experiments in the LHC was not able to detect the Higgs particle in a direct manner. Our reasoning is that in this picture the Higgs particle is a strongly interacting particle and need an infinite amount of energy to be isolated. Thus, according to this picture, the search of the Higgs has to be twisted to go the same way we feel the gluons.

A note to be mentioned is that this work does not give a final answer about the solution of the standard model problems like the Hierarchy and triviality problems. This is because, the Higgs mass receives other corrections from the vector Boson fields coupled to the Higgs field and the top quark contribution should also be taken into account. Taking this into account yield the result [21]

$$M_H^2 = M_0^2 - \frac{\Lambda^2}{8\pi^2 v^2} [M_H^2 + 2M_w^2 + M_Z^2 - 4M_t^2], \quad (18)$$

where M_0 is the bare mass contained in the unrenormalized Lagrangian. By the renormalization group analysis mentioned above, we made sure that the first term will be small as Λ

goes to higher values. For other terms, we know that all the masses in the standard model depend on the vacuum condensate which has been shown to have an exponential sharp decrease near $\lambda \rightarrow 0^+$ [14]. Accordingly, one may claim that the Higgs mass will stay protected against perturbations even for high energy scales. However, this claim should be tested in view of the renormalization group functions for the other sectors. In fact, this will take a substantial amount of time but it naturally becomes a target of our future work. The main aim we wanted by this work to spread the message that non-Hermitian and \mathcal{PT} -Symmetric ϕ^4 as now a physically acceptable model may help in the solution of the genuine existing problems in the standard model. These problems are well known to exist because the Hermitian Higgs mechanism used is both trivial and suffers from the gauge Hierarchy problem. Another important message that we need to mention is the that the vacuum energy of the non-Hermitian and \mathcal{PT} -Symmetric ϕ^4 is tiny and negative in $1 + 1$ dimensions compared to the Hermitian one [6, 22]. In another work [23], we showed that in $2 + 1$ dimensions, the vacuum energy is tiny as well as positive for a wide range of energy scales. In fact, these features are very interesting regarding the very false result of the prediction of the cosmological constant from quantum field theory which again is a manifestation of the gauge Hierarchy problem. In fact, positiveness of the vacuum energy is essential as it describes an expanding universe (negative pressure). We hope that using the asymptotically free non-Hermitian and \mathcal{PT} -symmetric ϕ^4 theory in the standard model will solve such genuine problems relying on the interesting features we explored above.

Since up till now the metric operator is not known, we tried to give the negative norm a physical meaning by considering the Higgs mass as a charge which can be positive or negative. We assert that, the form $\eta = \exp(-Q)$ can be obtained perturbatively using effective field representation. However, we did not prefer this direction as the resulting Hermitian Hamiltonian will be non-Renormalizable and thus incalculable.

A Higgs particle with negative mass makes sense in understanding how a potential bounded from above can have stable states. In this case, negative mass particles have the property of maximizing the classical action rather than the conventional positive mass particles which minimizing the classical action. In other words, a negative mass particle feels the bounded from above potential as the positive mass particle feels the bounded from below potentials.

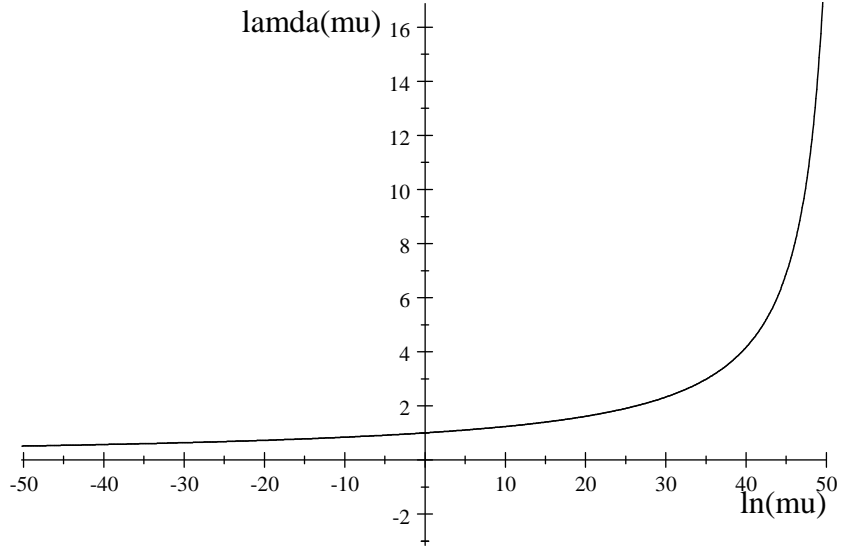


FIG. 1: The dependence of the renormalized coupling $\lambda(\mu)$ on the mass scale μ for the Hermitian ϕ^4 theory. In this figure, the theory is shown to be trivial for IR scales, while the coupling blows up for UV scales which means that the mass parameter will explode non-logarithmically at UV scales causing the Hierarchy problem.

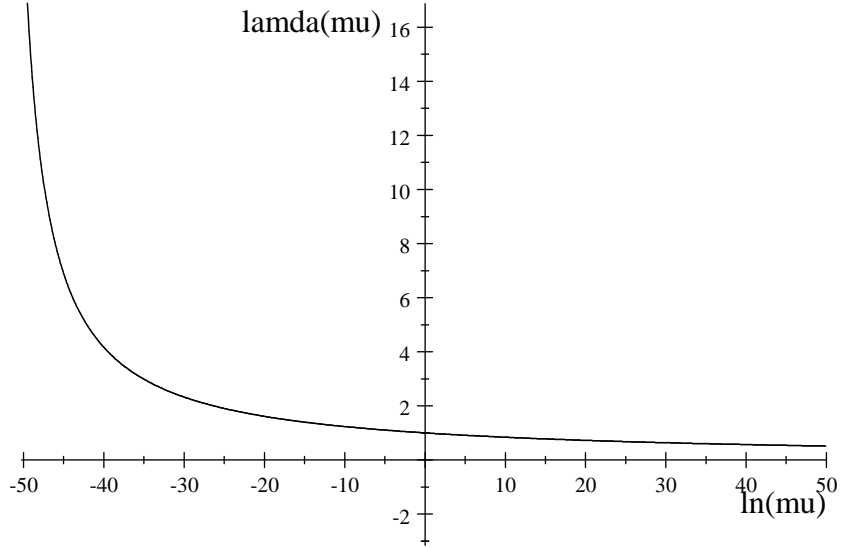


FIG. 2: The dependence of the renormalized coupling $\lambda(\mu)$ on the mass scale μ for the non-Hermitian ϕ^4 theory. One can realize that the theory is non-trivial as well as asymptotically free. Accordingly, the mass is finite for ultra UV scales.

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